Diploma Programme
Programme du diplôme
Programa del Diploma

# Mathematics
# Higher level
# Paper 3 – sets, relations and groups

Wednesday 18 November 2015 (afternoon)

1 hour

## Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[60 marks]**.



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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1.** [Maximum mark: 5]

Given the sets $A$ and $B$, use the properties of sets to prove that $A \cup (B' \cup A)' = A \cup B$, justifying each step of the proof.

**2.** [Maximum mark: 14]

The function $f : \mathbb{R} \to \mathbb{R}$ is defined as $f : x \to \begin{cases} 1 & , \ x \geq 0 \\ -1 & , \ x < 0 \end{cases}$.

(a) Prove that $f$ is

(i) not injective;

(ii) not surjective. [4]

The relation $R$ is defined for $a, b \in \mathbb{R}$ so that $aRb$ if and only if $f(a) \times f(b) = 1$.

(b) Show that $R$ is an equivalence relation. [8]

(c) State the equivalence classes of $R$. [2]

**3.** [Maximum mark: 10]

The set of all permutations of the elements $1, 2, \ldots 10$ is denoted by $H$ and the binary operation $\circ$ represents the composition of permutations.
The permutation $p = (1\ 2\ 3\ 4\ 5\ 6)(7\ 8\ 9\ 10)$ generates the subgroup $\{G, \circ\}$ of the group $\{H, \circ\}$.

(a) Find the order of $\{G, \circ\}$. [2]

(b) State the identity element in $\{G, \circ\}$. [1]

(c) Find

(i) $p \circ p$;

(ii) the inverse of $p \circ p$. [4]

(d) (i) Find the maximum possible order of an element in $\{H, \circ\}$.

(ii) Give an example of an element with this order. [3]

**4.** [Maximum mark: 18]

The binary operation $*$ is defined on the set $T = \{0, 2, 3, 4, 5, 6\}$ by
$a * b = (a + b - ab)(\bmod 7)$, $a, b \in T$.

(a) Copy and complete the following Cayley table for $\{T, *\}$. [4]

| $*$ | **0** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|
| **0** | 0 | 2 | 3 | 4 | 5 | 6 |
| **2** | 2 | 0 | 6 | 5 | 4 | 3 |
| **3** | 3 | 6 | | | | |
| **4** | 4 | 5 | | | | |
| **5** | 5 | 4 | | | | |
| **6** | 6 | 3 | | | | |

(b) Prove that $\{T, *\}$ forms an Abelian group. [7]

(c) Find the order of each element in $T$. [4]

(d) Given that $\{H, *\}$ is the subgroup of $\{T, *\}$ of order $2$, partition $T$ into the left cosets with respect to $H$. [3]

**Turn over**

**5.** [Maximum mark: 13]

A group $\{D, \times_3\}$ is defined so that $D = \{1, 2\}$ and $\times_3$ is multiplication modulo $3$.

A function $f : \mathbb{Z} \to D$ is defined as $f : x \mapsto \begin{cases} 1, & x \text{ is even} \\ 2, & x \text{ is odd} \end{cases}$.

(a) Prove that the function $f$ is a homomorphism from the group $\{\mathbb{Z}, +\}$ to $\{D, \times_3\}$. [6]

(b) Find the kernel of $f$. [3]

(c) Prove that $\{\mathrm{Ker}(f), +\}$ is a subgroup of $\{\mathbb{Z}, +\}$. [4]